**7.** (a) Find the value of $n$ so that the equations $V = r^n(3\cos^2\theta - 1)$ satisfies the relation $\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0$.

(b) If $x = e^r \cos\theta, y = e^r \sin\theta$ ; show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial\theta^2}\right)$.

## UNIT – IV

**8.** (a) Change the order of Integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate the same.

(b) Find by Triple Integration; the volume of sphere of radius 'a'.

**9.** (a) Find the surface of solid generated by revolving ellipse $x^2 + 4y^2 = 16$ about major axis.

(b) (i) Evaluate $\int_0^\infty e^{-x^2} x^{2n-1} dx$

(ii) $\int_0^{\pi/2} \sin^3 x \cos^{5/2} x\, dx$



*Roll No.* ............................

# 24002

# B. Tech 1st Semester (Common for All Branches) Examination – December, 2018

## MATHEMATICS – I

**Paper : Math-101-F**

*Time : Three Hours ]* *[ Maximum Marks : 100*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

***Note :*** Attempt ***five*** questions in all, by selecting ***one*** Question from each Unit. Question No. **1** is ***compulsory.***

**1.** (a) $\sum \cot^{-1} n^2$

(b) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

(c) Prove that the matrix :

$$\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \text{ is orthogonal.}$$

(d) If $f(x, y) = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$;. Find $\frac{\partial u}{\partial x}$.

(e) $u = \sin^{-1}(x-y); x = 3t, y = 4t^3$ find $\frac{du}{dt}$.

(f) Find the radius of curvature at the given following curve :

$$\sqrt{x} + \sqrt{y} = \sqrt{a} \quad at \left(\frac{a}{4}, \frac{a}{4}\right)$$

(g) Evaluate $\int_0^\infty x^3 e^{-x^3} dx$.

## UNIT – I

**2.** (a) Discuss the convergence of the series :

$$\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \ldots\ldots + \infty$$

(b) Discuss the convergence of the series :

$$x + \frac{2^2 . x^2}{2!} + \frac{3^3 . x^3}{3!} + \frac{4^4 . x^4}{4!} + \frac{5^5 . x^5}{5!} + \ldots\ldots\infty$$

**3.** (a) $\sum \frac{(x+2)^n}{3^n . n}$

(b) Discuss the convergence of an alternating series :

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots\ldots\ldots\ldots$$



## UNIT – II

**4.** (a) Find the inverse of matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ using elementary transformations.

(b) For what values of $a$ and $b$ do the equations $x + 2y + 3z = 6$, $x + 3y + 5z = 9$, $2x + 5y + az = 6$ have :

(i) No solution

(ii) Unique solution

(iii) More than one solution

**5.** (a) Are the following Vectors Linear Dependent ? If so, find the relation among them, $x_1 = (2, -1, 4), x_2 = (0, 1, 2), x_3 = (6, -1, 16)$.

(b) Find Eigen values and Eigen vectors of Matrix :

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

## UNIT – III

**6.** (a) Find the asymptotes of following curve :

$$3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$$

(b) Find the value of sin 46° correct to four places of decimals using Taylor series.