(b) If $U_n = \int_0^{\pi/4} \tan^n x \, dx$, show that $U_n + U_{n-2} = \frac{1}{n-1}$. Hence evaluate U_5.

Roll No. ....

91529

B. Sc. (Hons.) Physics 2nd Semester Latest Examination – April, 2018

MATHEMATICS-II

Paper : Phy-204

Time : Three Hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *two* questions from each Unit. All questions carry equal marks.

UNIT – I

1. (a) Show that the function defined by $f(x) = x^2$ is uniformly continuous in $[-2, 2]$.

(b) Show that the function $f(x) = \begin{cases} \sin\frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is discontinuous at $x = 0$.

2. (a) Expand tan x by Maclaurin's theorem as far as x^5 and hence find the value of tan 46°30' upto four decimal places.

(b) Show that $\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist.

3. (a) Show that the function $f : R^2 \to R$ defined by :

$$f(x,y) = \begin{cases} xy\frac{(x^2 - y^2)}{x^2 + y^2} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad \text{otherwise} \end{cases}$$

is continuous at (0, 0).

(b) State and prove Young's theorem.

4. (a) For the function :

$$f(x,y) = \begin{cases} \frac{1}{4}(x^2 + y^2)\log(x^2 + y^2) & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$$

show that $f_{xy} = f_{yx}$ for all x, y but the condition of Schwarz's theorem are not satisfied.

(b) Show that the function :

$$f(x,y) = \begin{cases} \frac{x^2 y^2}{x^4 + y^4}, & \text{for} \quad (x,y) \neq (0,0) \\ 0 \quad , & \text{for} \quad (x,y) = (0,0) \end{cases}$$

is not differentiable at the origin.

UNIT – II

5. (a) If a function f is defined on [0, a], $a > 0$ by $f(x) = x^3$, then show that 'f' is Riemann integrable on [0, a] and $\int_0^a f dx = a^4/4$.

(b) Prove that a continuous function f on [a, b] is integrable on [a, b]. Is the converse true ?

6. (a) Evaluate by summation $\int_a^b e^x dx$.

(b) Prove that every monotonic function is a integrable function.

7. (a) Prove that :

$$\frac{\pi^2}{9} \leq \int_{\pi/6}^{\pi/2} \frac{x}{\sin x} dx \leq \frac{2\pi^2}{9}$$

(b) State and prove Fundamental Theorem of Integral Calculus.

8. (a) Evaluate :

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$