

(b) Calculate $\int_C [(x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}] \cdot d\vec{r}$, where

C is the curve $y^2 = x$ joining (0,0) to (1,1). $3\frac{1}{2}$

(ii) State and prove Green's theorem. $3\frac{1}{2}$

(b) Verify Stoke's theorem for the function $\vec{f} = x^2\hat{i} + xy\hat{j}$ integrated around the square in the plane $z=0$ whose sides are along the lines $x=0, x=a, y=0, y=a$. $3\frac{1}{2}$

SECTION - V

(H) Define divergence of a vector function. 2

(h) Define Laplacian operator. 2

(o) Show that $\text{div}(\text{curl } \vec{f}) = 0$. 2

(d) State Stokes theorem. 2

(e) If $\vec{r} = \sin t \hat{i} + \cos t \hat{j} + t\hat{k}$ find $\left| \frac{d\vec{r}}{dt} \right|$. 2

(f) Prove that $\text{div} \cdot \vec{r} = 3$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 2

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B. Sc. (Pass Course) 2nd Semester Examination – May, 2019

MATHEMATICS (VECTOR CALCULUS)

Paper : 12BSM123

Time : Three hours / [Maximum Marks : 40

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Section - V is compulsory.

SECTION - I

1. (a) To show that : $3\frac{1}{2}$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \frac{1}{a \cdot (\vec{b} \times \vec{c})}$$

(b) If \vec{f} is a differentiable vector function of scalar variable t and $(\vec{f}) = f$ then prove $\frac{d}{dt}(\vec{f}^2) = 2f \frac{df}{dt}$ and $\vec{f} \cdot \frac{d\vec{f}}{dt} = f \frac{df}{dt}$. $3 \frac{1}{2}$

2. (a) If \vec{r} is a vector of a scalar t and $|\vec{r}| = r$ and \vec{a} is constant vector then differentiate w. r. t. t
 $\frac{\vec{r} + \vec{a}}{r^2 + a^2}$ $3 \frac{1}{2}$

(b) Find the unit tangent vector to the curve $\vec{r} = a(t - \sin t)\hat{i} + a(1 - \cos t)\hat{j}$ at any point t . $3 \frac{1}{2}$

SECTION - II

3. (a) Show that $\nabla f(\vec{r}) \times \vec{r} = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. $3 \frac{1}{2}$

(b) If \vec{f} is vector point function and ϕ is scalar point function having continuous second order partial derivative then prove $\nabla \times (\phi \vec{f}) = (\nabla \phi) \times \vec{f} + \phi(\nabla \times \vec{f})$. $3 \frac{1}{2}$

4. (a) Prove that curve $\left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5} (\vec{a} \cdot \vec{r})$

Where \vec{a} is constant vector. $3 \frac{1}{2}$

(2)

(b) Show that the function $\frac{1}{r}$, $|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$ is harmonic for provided $r \neq 0$.

SECTION - III

5. (a) If u, v, w are orthogonal curvilinear co-ordinates then $\frac{\partial \vec{r}}{\partial u}, \frac{\partial \vec{r}}{\partial v}, \frac{\partial \vec{r}}{\partial w}$ and $\nabla u, \nabla v, \nabla w$ reciprocal system of vectors.

(b) To show that $\nabla \cdot \vec{f} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} (h_2 h_3 f_1) + \frac{\partial}{\partial v} (h_3 h_1 f_2) + \frac{\partial}{\partial w} (h_1 h_2 f_3) \right]$ where $\vec{f} = f_1 \hat{e}_1 + f_2 \hat{e}_2 + f_3 \hat{e}_3$.

6. (a) If (r, θ, ϕ) are spherical Co-ordinates show $\nabla \phi = \nabla \times (r \operatorname{cosec} \theta \Delta \theta)$.

(b) Express the velocity and acceleration of a particle in spherical co-ordinates.

SECTION - IV

7. (a) Find the circulation of \vec{f} around the curve where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the curve $x^2 + y^2 = 1, z = 0$.

(3)

P.