8. (a) Sum of n terms the series $\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots\ldots$ and deduce the sum of $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$.

(b) Find the sum of $1 + \frac{\cos\alpha}{\cos\alpha} + \frac{\cos 2\alpha}{\cos^2\alpha} + \frac{\cos 3\alpha}{\cos^3\alpha}$

UNIT – V

(Compulsory Question)

9. (a) If n is an integer, show that $n(n^2 - 1)(3n+2)$ is divisible by 24.

(b) Find the remainder when 2^{20} is divided by 7.

(c) Evaluate $\mu(130)$.

(d) Evaluate $\left(\frac{19}{23}\right)$

(e) Prove that $i^i = e^{-(4n+1)\pi/2}$

(f) Solve for $x : \tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

Roll No. ....

21251

B. Sc. (Hons.) Mathematics 2nd Semester Examination – May, 2019

NUMBER THEORY AND TRIGONOMETRY

Paper : BHM-121

Time : Three hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **9** is ***compulsory***.

UNIT – I

1. (a) Show that a number is divisible by a iff the sum of its digits is divisible by 9.

(b) Show that there are infinitely many primes of the form $6n + 5$.

2. (a) Find the least the incongruent solution of $7x \equiv 5 \pmod{256}$.

(b) State and prove Wilson's theorem.

UNIT – II

3. (a) Solve the following set of congruences $2x \equiv 3$ (mod 5), $4x \equiv 2$ (mod 6), $3x \equiv 2$ (mod 7).

(b) Let m and n be positive integers. If every prime divisor of n is a preme divisor of m, then $\phi(mn) = n\,\phi(m)$.

4. (a) If $f(n) = \sum_{d/n} \mu(d)\, g\left(\frac{n}{d}\right)$ for every the integer n, then $g(n) = \sum_{d/n} f(d)$

(b) If $p \neq 3$ is an odd prime, show that

$$\left(\frac{3}{p}\right) = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod{12} \\ 1 & \text{if } p \equiv \pm 5 \pmod{12} \end{cases}$$

UNIT – III

5. (a) Show that the roots of $(1+x^3) = i(1-x)^3$ are $x = i \tan \frac{(4r+1)\pi}{12}$, where $r = 0,1,2$.

(b) Form an equation whose roots are $\sec\frac{\pi}{7}, \sec\frac{3\pi}{7}, \sec\frac{5\pi}{7}$. Hence show $\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7} = 4$.

6. (a) If z_1 and z_2 be complex quantities, show that

(i) $\cos z_1 + \cos z_2 = 2\cos\frac{z_1+z_2}{2}\cos\frac{z_1-z_2}{2}$

(ii) $\tan 3z_1 = \frac{3\tan z_1 - \tan^3 z_1}{1 - 3\tan^2 z_1}$.

(b) If $\tan(\theta + i\phi) = \sin(x + cy)$, prove that coth $\sin h^2 \phi = \cot x \sin 2\theta$.

UNIT – IV

7. (a) Resolve the following into real and Imaginary parts :

(i) $\log\cos(x + 1y)$

(ii) $\log(1 - c)$

(b) Separate $\tan^{-1}(x - ly)$ into real and imaginary parts.