SECTION – V

Show that the four points with position vectors $4\hat{i}+5\hat{j}+\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}, -\hat{j}-\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ are coplaner.

Prove that :

$$\frac{\vec{a}\times d^2b^{-1}}{dt^2}-\frac{d^2a^{-1}}{dt^2}\times\vec{b}=\frac{d}{dt}\left(\vec{a}\times\frac{d\vec{b}}{dt}-\frac{d\vec{a}}{dt}-\frac{d\vec{a}}{dt}\times\vec{b}\right)$$

If :

$\vec{a}=\sin\theta\,\hat{i}+\cos\theta\,\hat{j}+\theta\hat{k}$

$\vec{b}=\cos\theta\,\hat{i}-\sin\theta\hat{j}-3\hat{k}$

$\vec{c}=2\hat{i}+3\hat{j}-3\hat{k}$

find $\frac{d}{d\theta}\left[\vec{a}\times(\vec{b}\times\vec{c})\right]$ at $\theta=\frac{\pi}{2}$.

) Find λ,μ,v so that the vector :

$(2x+3y+\lambda z)\,\hat{i}+(\mu x+2y+3z)\,\hat{j}+(2x+vy+3z)\,\hat{k}$

is irrotational.

) Find curl (furl $\vec{f}$) of the function :

$\vec{f}=y(x+z)\hat{i}+z(x+y)+x(y+z)\hat{k}$.

) Define volume integral.

Roll No. ....

21253

B. Sc. (Hons.) Maths 2nd Semester Examination – May, 2019

VECTOR CALCULUS

Paper : BHM-123

Time : Three hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **9** (Section – **V**) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) Show that the vectors :

$\vec{a}-2\vec{b}+3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}$ and $\vec{a}-3\vec{b}+5\vec{c}$ are coplaner.

(b) Show that :

$$(\vec{b}\times\vec{c})\times(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})\times(\vec{b}\times\vec{d})+(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=-2\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{d}$$

P. T. O.

2. (a) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have a constant magnitude is $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$.

(b) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$, where t is time. Find the components of velocity and acceleration at $t = 1$ in the direction of $\hat{i} + \hat{j} + 3\hat{k}$.

SECTION – II

3. (a) For any vector $\vec{a}$, show that $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$, where $\vec{r}$ is the position vector of a point. Hence show that grad $\left[\vec{r}\ \vec{a}\ \vec{b}\right] = \vec{a} \times \vec{b}$.

(b) Given the curve of intersection of two surfaces $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$; Find the tangent line at the point (1, 0, 0).

4. (a) If div $(\phi(r)\vec{r}) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$, then prove that $\phi(r) = \frac{C}{r^3}$.

(b) Prove that :

$$\nabla \times (\vec{f} \times \vec{g}) = \vec{f}(\nabla \cdot \vec{g}) - \vec{f} \cdot (\nabla \vec{g}) + \vec{g} \cdot (\nabla \vec{f}) - \vec{g}(\nabla \cdot \vec{f})$$

SECTION – III

5. (a) If u, v, w are orthogonal curvilinear co-ordinates, then $\frac{\partial \vec{r}}{\partial u}, \frac{\partial \vec{r}}{\partial v}, \frac{\partial \vec{r}}{\partial w}$ and $\nabla u, \nabla v, \nabla w$ are reciprocal system of vectors.

(b) If (r, θ, ϕ) are spherical co-ordinates, sho[w] $\nabla \phi = \nabla \times (r \operatorname{cosec} \theta \nabla \theta)$.

6. (a) Express the velocity $\vec{v}$ and acceleration [...] particle in cylindrical co-ordinales .

(b) If p, ϕ, z are cylindrical co-ordinates, sho[w] $\nabla \phi$ and $\nabla \log \rho$ are solenoidal.

SECTION – IV

7. (a) A vector field is given by :

$$\vec{f} = \sin y\, i + x(1 + \cos y)\, \hat{j}$$

Evaluate the line integral $\int \vec{f} \cdot d\vec{r}$ along the c[...] given by $x^2 + y^2 = a^2, z = 0$;

(b) Evaluate $\iint \vec{f} \cdot \hat{n}\, ds$ where $\vec{f} = (x + y^2)\hat{i}$ [...] $+ 2yz\hat{k}$ and S is the surface of the pl[...] $2x + y + 2z = 6$ in the first octant.

8. (a) Verify divergence theorem for $\vec{f} = x^2\hat{i} + z\hat{j} +$ [...] taken over the cube bounded $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(b) Verify stoke's theorem [...] $\vec{f} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ when S is upper h[...] of the surface of sphere $x^2 + y^2 + z^2 = 1$ and C [...] its boundary.