(b) Prove that the infinite product $\prod_{n=1}^{\infty} \cos \frac{x}{2^n}$ converges to $\frac{\sin x}{x}$, where x is an arbitrary fixed non-zero number.

(a) Test the convergence a divergence of the series :

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots\ldots+\frac{1}{n}\right)\sin(n\theta+\alpha)$$

(b) State and prove Mertin's theorem.

UNIT – V

(a) Give an example to show that :

$$A^{\circ} \cup B^{B} \neq (A \cup B)^{\circ}.$$

) Find the derived set of $\left\{1+\frac{1}{2^n} \mid n \in N\right\}$.

) Discuss the boundedness of the sequence :

$$\left\langle \frac{2_n+3}{3_n+4} \right\rangle.$$

) State Logarithmic Test.

State Abel's test in arbitrary series.

Discuss the convergence of the infinite product :

$$\prod_{n=2}^{\infty} \left(1+\frac{(-1)^n}{\sqrt{n}}\right).$$

Roll No. ....

41251

B. Sc. (Hons.) Maths 4th Semester Examination – May, 2019

SEQUENCES AND SERIES

Paper : BHM241

Time : Three hours] ***[Maximum Marks : 60***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. **9** (Unit - V) is ***compulsory***. All questions carry equal marks.

UNIT – I

1. (a) Prove that between two distinct real numbers, there are infinitely many irrational numbers.

(b) Prove that $A^{\circ}$of a set A is the largest open subset of A.

2. (a) Prove that infimum of a set A are also infimum of $\overline{A}$ and contained in $\overline{A}$ as A is bounded below.

(b) State and prove converse of Heine Borel Theorem.

UNIT – II

3. (a) By definition, show that the sequence $\left\langle \frac{h^2+3h+5}{2n^2+5n+7} \right\rangle$ converges to $\frac{1}{2}$.

(b) Let $< a_n >$ be a sequence such that $a_n \neq 0 \; \forall n \in N$ and $\frac{a_n+1}{a_n} \to \rho$. If $|\rho| < 1$, then $\lim_{n \to \infty} a_n = 0$.

4. (a) Show that the sequence $<a_n>$ defined by $a_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots\ldots + \frac{1}{2^{n-1}}$ does not converge.

(b) Discuss the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}, x > 0$

(ii) $\sum_{n=1}^{\infty} \frac{x^n}{a^n + x^x}$

UNIT – III

5. Test for convergence of the following series :

(a) $\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \ldots\ldots\ldots$

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$

(c) $\frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \ldots\ldots\ldots\ldots$

6. (a) State and prove Gauss Test for the converge[nce of] an infinite series.

(b) Discuss the convergence of the following se[ries :]

(i) $\sum_{n=1}^{\infty} \left(n^{1+\frac{1}{n}} \right)^{-n}$

(ii) $\frac{a+x}{1!} + \frac{(a+2x)^2}{2!} + \frac{(a+3x)^3}{3!} + \ldots\ldots\ldots$

UNIT – IV

7. (a) Test the convergence and absolute convergence [of] the following series :

(i) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{(n+1)!}$

(ii) $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\sqrt{n^2-1} - n \right)$