(b) Find $f(x)$ if its Fourier sine transform is $\frac{s}{1+s^2}$.

(a) Using Parseval's Identity, prove that

$$\int_0^\infty \frac{x^2 dx}{(x^2+1)^2} = \pi/4.$$

(b) Solve $\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2}$ with Boundary conditions.

(i) $u = u_0$ when $x = 0$, $t > 0$ and the initial condition

(ii) $u = 0$ when $t = 0, x > 0$

UNIT– V

(a) Find the radius of convergence

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{5m}(x+1)^{3/m}$$

Show that $J_0''(x) = \frac{1}{x} J_1(x) - J_0(x)$

Find the value of $H_{2n}(0)$.

Find $L\left[\sin\frac{t}{2}\sin\frac{3t}{2}\right]$

$L^{-1}\left[\frac{s}{4s^2+15}\right]$

Find Fourier cosine transform of $e^{-5x}$.



Roll No. ............................

# 41252

## B. Sc. (Hons.) Maths 4th Semester Examination – May, 2019

## SPECIAL FUNCTIONS AND INTEGRAL TRANSFORMS

**Paper : BH242**

*Time : Three hours ]* *[ Maximum Marks : 60*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

***Note :*** Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. **9** (Unit – **V**) is ***compulsory***.

UNIT – I

1. (a) Find the power series solution of

$$(x^2-1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0,\ y(0)=2,\ y'(0)=3$$

(b) Solve :

$$x^2\frac{d^2y}{dx^2} + (x+x^2)\frac{dy}{dx} + (x-a)y = 0$$

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**2.** (a) Show that :

$$\frac{d}{dx}\left[J_n^2(x)\right]=\frac{x}{2n}\left[J_{n-1}^2(x)-J_{n+1}^2(x)\right]$$

(b) Find the solution of the following equations in terms of Bessel's function

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+4\left(x^2-\frac{n^2}{x^2}\right)y=0.$$

## UNIT– II

**3.** (a) Show that :

$$P_{2n}(0)=(-1)^n\frac{1.3.5......(2n-1)}{2.4.6.......2n}\;\frac{2n!}{(n!)^2(2^n)^2}$$

(b) Prove that :

$$\int_{-1}^{1} x\,P_n(x)\,P_{n-1}(x)\,dx=\frac{2n}{4n^2-1}$$

**4.** (a) To show that :

$$e^{2tx-t^2}=\sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(x)$$

(b) Show that :

$$\int_{-\infty}^{\infty} e^{-x^2}[H_n(x)]^2\,dx=2^n\,n!\int_{-\infty}^{\infty} e^{-x^2}\,dx=2^n\,n!\sqrt{\pi}$$





## UNIT – III

**5.** (a) Find the Laplace transform of the follow functions.

(i) $\cos^3 t$

(ii) $e^{-2t}\sin t\cos 3t$

(iii) $\dfrac{e^{-at}-e^{-bt}}{t}$

(b) Use Convolution theorem to evaluate :

$$L^{-1}\left(\frac{1}{(s+1)(s+9)^2}\right)$$

**6.** (a) Solve $\dfrac{d^2y}{dx^2}+\dfrac{2dy}{dx}+5y=e^{-t}\sin t$ by transfo method , where $y(0)=0,\ y'(0)=1$.

(b) Find the inverse Laplace transform of

(i) $\dfrac{1}{s^3(s^2+1)}$

(ii) $\dfrac{s}{s^4+4a^4}$

## UNIT – IV

**7.** (a) Find the Fourier transform of the function

$$f(x)=\begin{cases} x\,e^{-x}, & x>0 \\ 0 & x<0 \end{cases}$$



P. T. O.