B.Tech. 3rd Semester (CSE) (G-Scheme)
Examination, November-2023
MATHEMATICS-III
Paper - BSC-Math-203-G
(Multivariable Calculus ad Differential Equations)

Time allowed : 3 hours] *[Maximum marks : 75*

Note : Attempt any five questions in all, selecting one question from each Unit. Question no.1 is compulsory. All questions carry equal marks.

1. (a) If $f(x,y) = \frac{xy^2}{x^2+y^4}, (x,y) \neq (0,0)$.

   Show that $\underset{(x,y)\to(0,0)}{\text{Lt}} f(x,y)$ does not exist.

   (b) If $u = e^{xyz}$, prove that

   $$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1+3xyz+x^2y^2z^2)e^{xyz}.$$

   (c) Evaluate $\iint xy\,dxdy$ over the positive quadrant of the circle $x^2+y^2=a^2$.

   (d) Solve:

   $$(2xy+y-\tan y)\,dx+(x^2-x\tan^2 y+\sec^2 y)\,dy=0.$$

(e) Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 0$.

(f) Solve the equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters. $6 \times 2.5 = 15$

Unit-I

2. (a) Find the value of n, so that the equation

$V = r^n (3\cos^2\theta - 1)$ satisfies the relation

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0.$$

(b) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, prove that,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u.$$

$2 \times 7.5 = 15$

3. (a) Examine for minimum and maximum values:

$\sin x + \sin y + \sin (x+y)$.

(b) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. $2 \times 7.5 = 15$

Unit-II

4. (a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dydx$ by changing the order of integration.

(b) Evaluate $\iiint z(x^2+y^2+z^2)\, dxdydz$, through the volume of the cylinder $x^2+y^2=a^2$ intercepted by the planes $z=0$ and $z=h$. 2×7.5 = 15

5. (a) Show, by double integration, that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3}a^2$.

(b) Evaluate $\int_0^1 \int_0^{x^2} \int_0^{x+y} (x-2y+z)\, dzdydx$.

2×7.5 = 15

Unit-III

6. (a) Solve $(x^2+1)\frac{dy}{dx}+4xy=\frac{1}{x^2+1}$.

(b) Solve $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$.

2×7.5 = 15

7. (a) If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60° C in 12 minutes, find the temperature of the body after 24 minutes.

(b) Find the orthogonal trajectory of the family of the curve $r^2=a^2\cos 2\theta$. 2×7.5 = 15

Unit-IV

8. (a) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$.

(b) Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$.

$2 \times 7.5 = 15$

9. (a) Solve the simultaneous equations:

$\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$; given that $x(0) = 2, y(0) = 0$.

(b) An e.m.f. E sin pt is applied at t = 0 to a circuit containing a capacitance C and inductance L. The current *i* satisfies the equation $L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E \sin pt$. If $p^2 \frac{1}{LC}$ and initially the current *i* and the charge *q* are zero, show that the current at time t is $\frac{Et}{2L} \sin pt$, where $i = \frac{dq}{dt}$. $2 \times 7.5 = 15$